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(February 1, 2001)

Based on lattice data on quark number susceptibility (QNS) measured as a function of temperature, we propose that an *induced* flavor gauge symmetry phrased in terms of hidden local symmetry (HLS) is continuously connected to the *fundamental* color gauge symmetry of QCD at the chiral phase transition. We exploit for this purpose the recent developments on color-flavor locking (CFL) in QCD and Harada-Yamawaki's "vector manifestation" of chiral symmetry formulated in hidden gauge symmetry theory. It is argued that BR scaling can be naturally fit into the scenario that combines the CFL and the HLS.

It was observed in [1] that the quark number susceptibility (QNS) $\chi_\pm = (\partial/\partial\mu_u \pm \partial/\partial\mu_d)(\rho_u \pm \rho_d)$ where $\rho_{u,d}$ and $\mu_{u,d}$ are, respectively, u, d-quark number density and chemical potential measured on lattice as a function of temperature [2,3] exhibited a smooth and rapid changeover from a flavor gauge symmetry or hidden gauge symmetry to QCD color gauge symmetry at the chiral transition temperature T_c . It was suggested there that at the phase transition, the flavor gauge symmetry – which is induced and hence not fundamental – gets *converted* to the color gauge symmetry – which is fundamental, implying that they could be related in a direct albeit intricate way. In this Letter, we suggest how this can be realized in terms of color-flavor-locked (CFL) quark-antiquark and diquark condensates and "vector manifestation" of chiral symmetry. We shall also discuss how BR scaling [4] can fit into the general scheme that results from these developments.

Our argument relies on two recent developments that come from seemingly unrelated sectors. One is the suggestion by Harada and Yamawaki [5] that the phase transition from the Nambu-Goldstone phase to the Wigner-Weyl phase involves "vector manifestation" of chiral symmetry which states in the context we are interested in that at the phase transition, the longitudinal components of the light-quark vector mesons (i.e., the triplet ρ in the 2-flavor case) and the triplet pions (π^a) come together becoming massless in the chiral limit. The massless vectors decouple à la Georgi's vector limit [6] with $g_V = 0$ and $a = 1$ where g_V is the hidden gauge coupling and a is the parameter that locks the left and right symmetries of chiral symmetry for $a \neq 1$. The other important development is the proposal by Berges and Wetterich [7] that color and flavor (isospin) get completely locked by the quark-antiquark condensate in the color-octet ($\mathbf{8}$) channel

$$\chi = \langle \bar{q}^a_\alpha \sum_{i=1}^{3} (\tau_i)_{\alpha\beta} (\lambda_i)^{ab} q^b_\beta \rangle \tag{1}$$

and the diquark condensate in the color-antitriplet ($\bar{\mathbf{3}}$) channel

$$\Delta = \langle q^a_\alpha (\tau_2)_{\alpha\beta} (\lambda_2)^{ab} q^b_\beta \rangle. \tag{2}$$

In (1) and (2), the indices α, β denote the flavors and a, b the colors.

Before going into our main thesis, we briefly summarize the essential results of the two developments in the language that is best suited for our purpose.

- **Color-flavor locking**

In [7], considering the case of two light flavors, Berges and Wetterich argue that both the χ and Δ condensates can be nonzero in the vacuum. The color is then completely broken instead of the partial breaking that takes place if $\Delta \neq 0$ and $\chi = 0$ [8]. As a consequence, all octet gluons and six quarks become massive by the Higgs mechanism, and three Goldstone pions get excited. All of the excitations are integer-charged. Among the 8 massive gluons, three of them are identified with the isotriplet ρ's with mass

$$m_\rho = \kappa g_c \chi \tag{3}$$

where κ is an unknown constant and g_c the color gauge coupling. The fourth vector meson is identified with the isosinglet ω with mass

$$m_\omega = \kappa' g_c \Delta \tag{4}$$

where κ' is another constant. The remaining four vector mesons have exotic quantum numbers and are presumably heavy and decouple from low-energy regime. We will not be concerned with them here. As for the fermions, there are two baryons with the quantum numbers of the proton and neutron with their masses proportional to the scalar condensate ϕ,

$$\phi = \langle \bar{q}^a_\alpha q^a_\alpha \rangle. \tag{5}$$

The four remaining fermions are also of exotic quantum numbers with zero baryon number and heavier, so we assume that they also decouple from the low-energy sector we are interested in. What concerns us in this Letter is therefore the three pions, the proton and neutron, the ρ-mesons and the ω-meson.

• Vector manifestation of hidden local symmetry

We consider the hidden local symmetry (HLS) theory of Bando et al [9] with the symmetry group $[U(2)_L \times U(2)_R]_{global} \times [U(2)_V]_{local}$ consisting of a triplet of pions [14], a triplet of ρ-mesons and an ω-meson. For simplicity we are putting the ρ and ω into an $U(2)$ multiplet. This is motivated by the observation that in the vacuum, they are nearly degenerate and the quartet symmetry is fairly good. In this theory, baryons (proton and neutron) do not appear explicitly. They can be considered as having been integrated out. If we wish, we can recover them as solitons (skyrmions) of the theory. At present, it is not known how to consistently incorporate the baryons in HLS theory and hence treating density or baryon chemical potential is problematic (see [10]). Here we shall consider the effect of temperature and make conjectures on density effects.

The relevant degrees of freedom are the left and right chiral fields denoted by $\xi_{L,R}$ and the hidden local gauge fields denoted by $V_\mu \equiv V_\mu^\alpha T^\alpha = \frac{\tau^\alpha}{2}\rho_\mu^\alpha + \frac{1}{2}\omega_\mu$ with $\mathrm{Tr}(T^\alpha T^\beta) = \frac{1}{2}\delta^{\alpha\beta}$. If we denote the $[U(2)_L \times U(2)_R]_{global} \times [U(2)_V]_{local}$ unitary transformations by (g_L, g_R, h), then the fields transform $\xi_{L,R} \mapsto h(x)\xi_{L,R}g_{L,R}^\dagger$ and $V_\mu \mapsto h(x)(V_\mu - i\partial_\mu)h^\dagger(x)$. In the vacuum, the local gauge symmetry is spontaneously broken and the vector mesons get the Higgs mass

$$m_\rho = m_\omega = a g_V f_\pi \tag{6}$$

where g_V is the *flavor gauge* coupling constant, f_π is the pion decay constant related to the condensate ϕ and a signals that the $U(2)\times U(2)$ symmetry is spontaneously broken by taking a value $a \neq 1$. In nature $a \approx 2$ corresponds to the KSRF relation for the ρ mesons. In the vacuum, the ρ and ω are nearly degenerate satisfying the mass formula (6). It may be that the $U(2)$ symmetry is broken in medium but in this paper we shall simply assume that it continues to hold in hot and/or dense matter up to the chiral transition point. Harada and Yamawaki [11–13] have shown that the theory has an ultraviolet fixed point $g_V = 0$ and $a = 1$ which corresponds to Georgi's vector limit [6]. Since the HLS theory is an effective theory with a cutoff at the chiral scale Λ_χ, the fixed point should correspond to the *bare* theory at the cutoff scale. Harada and Yamawaki [5] then argue, using Wilson's renormalization-group reasoning, that the chiral restoration should then be signaled by $f_\pi = 0$ together with $g_V = 0$ and $a = 1$. This means by the mass formula (6) that at the phase transition, the vector mesons become massless with both g_V and f_π going to zero and decouple. The longitudinal components of the vector mesons turn into a quartet of massless scalars, i.e., scalar Goldstone bosons that are the chiral partners of the pseudoscalar Goldstone bosons, π^i (and "η" [14]). This is the vector manifestation of chiral symmetry. This scenario is distinct from the "standard," though as yet unestablished, picture in which the ρ and a_1 come together as do the pions and a scalar σ. In the standard scenario, there is nothing which forces the vector to become massless and decouple. They can even become more massive at chiral restoration than in the vacuum [15]. Thus the vanishing of the vector-meson mass is a *prima facie* signal for the phase transition in the Harada-Yamawaki picture [16].

Harada and Yamawaki did not show that chiral restoration must necessarily occur in the vector manifestation at high temperature and/or density. What they showed is that such a phase transition can naturally take place at some large number of flavors $N_f \gtrsim 4$, in consistency with what is expected in large N_f QCD [17]. Though it has not yet been shown either for high temperature or for high density, we will assume that the phase change is a generic feature and that apart possibly from the order of transition, chiral restoration does occur in the same manner *independent* of what drives the phase change.

• Implications of the lattice QNS

We shall now exploit the results of the lattice calculation of quark number susceptibility (QNS) to make a link between the color-flavor locking and the HLS summarized above.

We have argued in [1] that the "measured" singlet and non-singlet QNS's [2] indicate that both the ρ and ω couplings vanish at the transition temperature T_c and that this is consistent with that HLS coupling $g_V \to 0$ faster than $f_\pi \to 0$ as $T \to T_c$. This implies via (6) that the ρ and ω become massless and decouple simultaneously. Now the color-flavor-locking involves two condensates χ and Δ. Symmetry arguments alone do not provide a relation between the two. However if one accepts the HLS scenario with the phenomenologically motivated $U(2)$ symmetry, then (3) and (4) imply that

$$\kappa\chi \propto \kappa'\Delta \tag{7}$$

with both χ and Δ going to zero at $T \approx T_c$. It is therefore reasonable to conjecture that the mass formulas (3), (4) and (6) – all of which are Higgsed – are related

$$a g_V f_\pi \sim \kappa g_c \chi \sim \kappa' g_c \Delta. \tag{8}$$

Next, we have shown in [1] that above the chiral transition temperature $T \gtrsim T_c$, the QNS's can be well described by perturbative gluon exchange with a gluon coupling constant $\frac{g_c^2}{4\pi} \approx 0.19$ and argued that the flavor gauge symmetry *cedes* to the fundamental QCD gauge symmetry. Now the HLS theory is moot on what it could be beyond the chiral restoration point since the theory essentially terminates at T_c. We propose that this is where the color-flavor locking of [7] phrased in the QCD variables takes over by supplying a logical language for crossing-over from below T_c to above T_c. Indeed (8) describes the *relay* that must take place in terms of the hidden flavor gauge coupling g_V on one side and the color gauge coupling g_c on the other side.

What we can say from the lattice results is that as temperature nears the critical from below, both condensates χ and Δ must melt in the sense of Harada-Yamawaki's vector manifestation. Above T_c, the color and flavor must unlock, with the gluons becoming massless and releasing the scalar Goldstones. The dynamics of quarks and gluons in this regime will then be given by hot QCD in the proper sense. Now how the two condensates melt as temperature is increased is a dynamical question which seems to be difficult to answer unambiguously with models. It will have to be up to lattice measurements to settle this issue. Our chief point here – which is of course a conjecture in the absence of a solid proof but plausible in connection with BR scaling and the lattice data – is that their melting is intricately connected.

- **Density effects and BR scaling**

The situation is quite different in dense medium. There is no guidance from lattice since it is impossible at present to put density on lattice except for the unphysical cases of two colors or adjoint quarks. Certain models indicate that the phase structure near chiral restoration could be quite involved and complex. As suggested by Schäfer and Wilczek [18], an intriguing possibility is that the three-flavor color-flavor locking continues all the way down to the "chiral transition density" (ρ_c) – whatever it may be – in which case there will be no real phase change since there will then be a one-to-one mapping between hadrons and quark/gluons, e.g., in the sense of "hadron-quark continuity." However the non-negligible strange-quark mass is likely to spoil the ideal three-flavor consideration. One possible alternative scenario is that viewed from "bottom-up," one gets into the phase where $\chi = \sigma = 0$ and $\Delta \neq 0$ corresponding to the two-flavor color superconducting (2csc) phase [8]. Unless Δ goes to zero at ρ_c, this would mean that the ρ mesons become massless but the ω meson remains massive. One cannot say that this is inconsistent with the vector manifestation since the HLS does not require that $U(2)$ symmetry hold at the chiral restoration point or in medium in general. On the other hand, it is equally possible that both χ and Δ approach zero from below ρ_c and then Δ picks up a non-zero value at or above ρ_c in which case we will preserve the mass formula (6) as one approaches ρ_c. This is the most plausible scenario that we favor in the discussion that follows.

Among the scaling relations implied by BR scaling [4,1], the one most often referred to is the dropping of the ρ-meson mass in medium. This relation has been extensively discussed recently in connection with the CERN-CERES data on dilepton production in heavy-ion collisions. The simplest explanation for the observed dilepton enhancement at an invariant mass ~ 400 MeV is to invoke BR scaling for the excitations relevant in the process [19]. It turns out however that this explanation is not unique. One could explain it equally well if the ρ meson "melted" in dense medium with a broadened width [20]. Since the process is essentially governed by a Boltzmann factor, all that is needed is the shift downward of the ρ strength function: the expanding width simply does the job as needed for the dilepton yield. If one calculates the current-current correlation function in low-order perturbation theory with a phenomenological Lagrangian, it is clear, because of the strong coupling of the ρ meson with the medium, that the meson will develop a large width in medium and "melt" at higher density. The upshot of the dilepton experiments then is that they cannot distinguish the variety of scenarios that probe average properties of hadrons in the baryon density regime – which is rather dilute – encountered in the experiments. As has been argued by the authors of Ref. [21], photons from Pb-Pb collisions at CERN-SPS energy could, however, distinguish the two mechanisms since while insensitive to the collisional broadening of the vector mesons, they are affected by BR scaling. It was found there that the photon spectrum is consistent with BR scaling but not with the melting of the vector ρ.

If the connection developed above between the CFL and the HLS is accepted, then it is clear that the masses of the vector mesons *must drop sharply* when the density is near the critical. If the mass drops, then the width should become narrower, with the meson becoming more a quasiparticle at high density than at lower density. *This is the underlying picture of BR scaling.* This picture is further strengthened in [10] by a "sobar-model" involving gluons (in analogy to ρ-sobar) using a language familiar in nuclear physics. Now what about the matter that is not so dense, say, near nuclear matter density? By far the most clear-cut indication for BR scaling comes from observations in nuclei, that is, up to nuclear matter density. In nuclei, the evidence is mostly indirect but the data are more precise and hence more quantitative. Several cases evidencing BR scaling are discussed in a recent review [10]. Some are somewhat model-dependent and hence subject to objections. The most direct case is the $(e, e'p)$ response functions in nuclei [22,10] where the effect of BR scaling is more convincingly exposed.

Here we mention one case which is interesting conceptually but little appreciated by the physics community. It has to do with the fact that nuclear matter owes its stability to a Fermi-liquid fixed point. Certain interesting nuclear properties are then calculable in terms of the Fermi-liquid fixed point parameters [23]. Among others, it has been shown [23] that the Landau parameter F_1 can be expressed in terms of the BR scaling factor $\Phi(\rho) \equiv m_\rho^\star(\rho)/m_\rho(0)$. A well-established example is the anomalous gyromagnetic ratio δg_l in nuclei which is found to be given by

$$\delta g_l = \frac{4}{9}\left[\Phi^{-1} - 1 - \frac{1}{2}\tilde{F}_1(\pi)\right]\tau_3 \tag{9}$$

where $\tilde{F}_1(\pi)$ is the pionic contribution to F_1 which is completely given for any density by chiral symmetry. At $\rho = \rho_0$, it takes the value $\tilde{F}_1(\pi)|_{\rho=\rho_0} = -0.153$. Note that (9) depends on only one parameter, Φ. This parameter can be extracted either from nuclear matter saturation ($m_\omega^\star$) or from Gell-Mann-Oakes-Renner formula for in-medium pion ($f_\pi^\star$) or from a QCD sum rule for the ρ meson ($m_\rho^\star$). All give about the same value, $\Phi(\rho_0) \approx 0.78$. Given Φ at nuclear matter density, Eq.(9) makes a pleasingly neat prediction,

$$\delta g_l = 0.23\tau_3. \tag{10}$$

This was confirmed by a measurement in the Pb region [24],

$$\delta g_l^p = 0.23 \pm 0.03. \tag{11}$$

We should point out that this is actually a test for the scaling of the nucleon mass $m_N^\star/m_N \approx \sqrt{g_A^\star/g_A}\Phi$, and *not directly* of the vector meson mass. Nonetheless, it does verify BR scaling since the Φ is extracted from $m_\rho^\star$ or $f_\pi^\star$ with the assumption that the BR scaling holds as a whole.

We have thus far argued that the vector mesons exhibit BR scaling at near nuclear matter density and their mass must vanish – in chiral limit – at ρ_c in accordance with the vector manifestation. While we can say nothing as to what happens at densities more dilute than nuclear matter density, we can however conclude that BR scaling must continue to hold in $\rho_0 \lesssim \rho \lesssim \rho_c$.

There is one serious obstacle to our proposal that needs to be cleared up before it can be considered as a truly viable one and that is the exotic bosons and fermions that figure in the Berges-Wetterich scheme. Our connection between the CFL and HLS pictures relies crucially on that at the chiral restoration point T_c or ρ_c , all three condensates χ, Δ and ϕ go to zero. The only natural way this can happen is if the three condensates are directly related. A possible consequence of this on the exotic states is that near the critical point, they also become light – and massless in the chiral limit – since they are all proportional to the three condensates in various combinations. These states are not accommodated in the HLS theory and therefore a direct connection will be broken. The only way that the connection can remain intact is if the exotic states get a large mass from other sources that remain as one goes across the transition point or they remain decoupled.

We are grateful for discussions with, and comments from, Masayasu Harada and Koichi Yamawaki. The work of GEB was supported by the US Department of Energy under Grant No. DE-FG02-88 ER40388.

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